connection with formal applications of group theory to atomic spectra.[20] With this new choice of Eulerian angles, there is no catastrophe when the moments of inertia become equal. The difficulty is rather that the anomalous coefficients, now different from (18), show an undue preference for one particle. In fact, the situation is analogous to that upon which we commented after Eq. (7) in the two-dimensional example, where we chose the coordinate φ_1 to specify the radius vector of one particle. The second order perturbation calculation, of course, removes the dissymmetry, and restores the conventional result (1). This calculation might not be easy to make explicitly, due to the clumsy, unsymmetrical nature of the coordinates, but can only lead to (1), at least when the moments of inertia are unequal, since we have proved by means of Eckart's coordinates that (1) then is inevitable. With the unsymmetrical coordinates, there is nothing to single out the case of equal moments of inertia, so that (1) must remain valid in the limit of equality. This argument furnishes the safest way to see that the terms of the third degree in P_1, P_2, P_3 neglected in our perturbation calculation extending to the second order, are not unduly important when the moments of inertia nearly coincide. This result is not obvious when the symmetrical coordinate system is used.

We may remark incidentally, that it is only when two or more moments of inertia coincide that the expressions ζ_i involved in (1) can have non-vanishing values, for otherwise the molecule will not have sufficient symmetry to permit the vibrational degeneracy requisite for $\zeta_i \neq 0$.

The writer wishes to thank Professor Carl Eckart for interesting discussion and correspondence.

[20] E. Wigner, Zeits. f. Physik **43**, 624 (1927).

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On the Plasticity of Crystals

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In the following, a theory is given with the purpose of establishing a mathematical relation between the stress and the strain in a crystal when plastically deformed. The existence of a "secondary structure" in crystals is adopted as a basic hypothesis. This structure was pointed out by Professor F. Zwicky to be a consequence of what he calls "cooperative phenomena." The assumption that gliding in crystals takes place between the blocks of the secondary structure is the starting point of the following theory. The additional hypothesis of assuming a statistical distribution of the different forces which produce gliding between the blocks, gives us the means for going further in the calculations. The final result which is the stress strain curve of a crystal, is an exponential law containing three constants, i.e., the torsional modulus G, the elastic limit, (γ_s, τ_s) and the maximum applicable stress τ_m. The form of the hysteresis cycles is deduced from the same considerations and moreover a formula is obtained for the areas of the cycles. Experimental verifications were made on a single crystal of copper, and also on ordinary microcrystalline copper.

PART I

IT is well known that when the yield point is reached in a crystal, gliding takes place between some of the crystallographic planes. If we plastically deform a crystal through application of a sufficiently high stress in a given direction, and then reverse the stress, the gliding is reversed also, but we never obtain the initial state. Plastic deformation of a crystal at low temperatures in general leads to "cold hardening." The conception of the ideal crystal, defined by a geometrical arrangement of particles in a lattice, cannot explain the phenomenon of cold hardening. Zwicky[1-6] has studied other physical properties of crystals which are incompatible with the idea of an ideal lattice. He comes to the conclusion that these properties may be understood in terms of a "secondary structure" in crystals. In this discussion we admit the existence of this secondary structure, and, following Zwicky, we denote as π-planes the crystallographic planes characterizing the secondary lattice. Gliding

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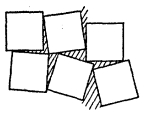

FIG. 1. Model of plastically deformed crystal after Zwicky.

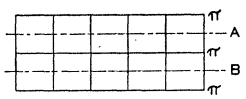

FIG. 2.

occurs in these planes for shearing stresses far smaller than these derived from the theory of ideal lattices.

The laws governing gliding between two planes are not known very accurately. Two different theories were proposed by Frenkel[7] and Becker[8] with the view of defining the conditions under which gliding takes place. Neither of them gives any physical explanation of cold hardening.

Experimentally we know that gliding increases with the shearing stress following a regular curve. In this connection aluminum crystals were studied with very great care by Taylor.[9]

Schematically the pseudostable configuration of a plastically deformed cubic crystal can be represented as shown in Fig. 1. (Rotation of the blocks is exaggerated.) Deformation has taken place along the planes and the blocks have different relative orientations. The shearing stress which is necessary to produce a new gliding is visibly increased (cold hardening). Let us consider a part of a crystal consisting of two slices of elementary secondary blocks (Fig. 2). Let us call A and B the two planes of symmetry of each one of the slices. These two planes are at a distance equal to the constant of the secondary lattice. If we apply a tangential force between A and B great enough to produce gliding along the π-plane, the blocks are disturbed and the configuration becomes that represented by Fig. 1. From the level A to the level B the force is transmitted in two different ways. From A to π, the transmission is purely elastic; along π we must consider a friction between the blocks; from π to B, we have again an elastic transmission. We may assume that the *translation* of each block is the same. On the contrary, the *rotation* is different, and the force required to produce a gliding between two corresponding blocks is consequently *different.*

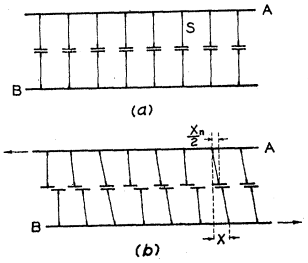

FIG. 3. Mechanical model showing the way the force is transmitted between the blocks of the "secondary structure."

To depict the phenomenon of plastic deformation we consider the mechanical *model* illustrated in Fig. 3. The two planes A and B previously mentioned are represented by two parallel plates to which are attached a series of springs S. These springs are connected by small plates. These can glide on each other. Let us assume that the elastic constants of the springs are all the same, but that the forces required to produce gliding of the surfaces are different. These hypotheses are suggested by the correspondence between the crystal and the model. If we displace the two plates A and B in such a way that they remain essentially parallel, the tangential force will be determined by the values of the elastic constants of the springs and the different forces of each gliding element. Let x be the displacement of the plane A with respect to the plane B. Consider n elements per unit length. To each element corresponds a parameter f_n, which is the minimum value of the force which must be applied to obtain a gliding. It is more convenient in computation to consider the elastic displacement x_n corresponding to this force f_n, as given by the relation $f_n = Kx_n$, K being the constant of the spring. The amplitude of the gliding is, for each element equal to the displacement $x - x_n$, x being the displacement which we impose on every element. Let us divide the interval x into small intervals Δx. In each of these intervals, we have a given number Δn of elements which begin to glide; we have a correspondance between the Δn and the Δx. If we divide the deformation into intervals small enough, we can assume that the correspondance between the ratio dn/dx and x

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[8] R. Becker, Physik. Zeits. **26**, 919 (1925).
[9] G. Taylor, Proc. Roy. Soc. **A102**, 650 (1923); **A111**, 533 (1926).

is given by a continuous curve. If $dn/dx=\varphi(x)$ is the equation of this curve, the number of elements for which gliding occurs between x and $x+dx$ is given by $dn=\varphi(x)dx$. In the same way, the number of elements for which gliding occurs when the deformation increases from x_1 to x_2 is given by the integral

$$n_{12}=\int_{x_1}^{x_2}\varphi(x)dx.$$

Knowing the analytical form of the function $\varphi(x)$, we are able to compute, as we shall see later, the total force transmitted by all the elements for a given value of the deformation.

The form of the function $\varphi(x)$ is to some extent arbitrary, and the choice of this function is the main hypothesis on which the following computations are based. We suppose that the number dn of elements which glide between x and $x+dx$, is given by an exponential law of the form

$$dn=Ae^{-Bx}dx. \tag{1}$$

We can justify the exponential law by a very simple argument which is used in the kinetic theory of gases to introduce the notion of the mean free path of a particle. Let us consider a *total* displacement x and denote by $P(x)$ the probability of an element retaining an *elastic* displacement x. Suppose that the probability of obtaining gliding for an elementary total displacement dx is proportional to dx. If a is the constant of proportionality, the probability of having no gliding in the length dx is $1-adx$. If we apply the theorem of combined probabilities we have the relation

$$P(x+dx)=P(x)[1-adx].$$

The Taylor expansion theorem limited to the two first terms gives

$$dP(x)/dx=-aP(x).$$

If b denotes a constant, we have $P(x)=be^{-ax}$. The constant b is equal to 1 because the function $P(x)$ must be unity when $x=0$. Consequently $P(x)=e^{-ax}$. Let n be the total number of elements per unit length. The number dn of these having a gliding between x and $x+dx$ will be

$$dn=ne^{-ax}-ne^{-a(x+dx)},$$

since ne^{-ax} is the number of elements which do not glide during the displacement x, and $ne^{-a(x+dx)}$ is the number of elements which do not glide during the displacement $x+dx$. If we develop the exponential function, we obtain

$$dn=nae^{-ax}dx. \tag{2}$$

By definition, the factor a has the dimension of the inverse of a length. Let us put

$$L=1/a.$$

The formula (2) becomes

$$dn=(n/L)e^{-x/L}dx \tag{3}$$

in which L is a length.

It is now possible to calculate the force which we must apply between the two planes A and B (Fig. 3) in order to produce a given deformation x_1. The elements of the mechanical model (or the blocks of the secondary structure in the crystal) may be divided into two classes, depending upon the value of the deformation x_1. In the first class we put those elements for which the gliding occurs between the two surfaces during the increase of the deformation from 0 to x_1. In the second class are put the elements whose deformation is purely elastic. We shall calculate separately the force corresponding to those two classes and the sum will be the total force.

Each element of the first class takes a different part in the transmission of the force. The element for which gliding occurred when the deformation was x transmits a force equal to Kx. This force remains constant as the deformation increases up to the value x_1. If we multiply the number dn given by Eq. (3) by the force Kx, we obtain the elementary force corresponding to the elements dn, i.e.,

$$dF_1=Kx(n/L)e^{-x/L}dx.$$

The total force will be obtained by an integration from 0 to x_1.

$$F_1=(Kn/L)\int_0^{x_1}xe^{-x/L}dx. \tag{4}$$

In the second class we consider the elements for which the deformation is essentially elastic. All those elements, when the deformation is x_1, transmit the same force Kx_1. The same argument

as before gives us the elementary force

$$dF_2 = Kx_1(n/L)e^{-x/L}dx,$$

and the total force will be

$$F_2 = (Kn/L)x_1\int_{x_1}^{\infty} e^{-x/L}dx. \tag{5}$$

If we combine Eqs. (4) and (5), we obtain the total force per unit length transmitted by the elements of the two classes,

$$F = -KnL[1 - e^{-x_1/L}].$$

The minus sign means that the force is in the direction opposite to that taken as positive for the deformation. In the following, we shall not take this sign into consideration.

We can replace our linear model by a two-dimensional one in which we consider n^2 elements per unit area. The argument is exactly the same, and we have the formula

$$F = Kn^2L[1 - e^{-x_1/L}]. \tag{6}$$

Let us denote $\gamma = x/D, \quad l = L/D, \tag{6'}$

in which D is the constant of the secondary lattice. These ratios are without dimension and the first one corresponds to the definition of a shearing strain in a solid. With those notations, Eq. (6) becomes

$$F = Kn^2Dl[1 - e^{-\gamma/l}].$$

By definition, F, being the force per unit area, is the stress. We call it τ. On the other hand, the factor Kn has the dimensions of a modulus of elasticity; let us call it G. The product Dn is obviously unity. Thus we have the final formula

$$\tau = Gl[1 - e^{-\gamma/l}]. \tag{7}$$

The slope of the tangent at the origin of this exponential curve (Fig. 4) is G. The asymptote

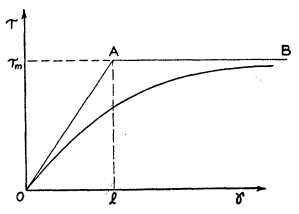

FIG. 4. Form of the plastic stress strain curve.

has an ordinate Gl. Let us denote this ordinate by

$$\tau_m = Gl. \tag{8}$$

Hence τ_m is the maximum stress that can be applied without producing rupture of the crystal. We can calculate the mean value of the elastic displacement. This mean value is given by the formula

$$(1/n)\int_0^{\infty} xdn.$$

If we replace dn by its value (3), we find that this mean value is precisely L. We suggest to call "average element" the element having L as elastic displacement. Eqs. (6) and (8) show us that the two lines OA and AB (Fig. 4) are the stress strain diagram of this average element. Hence complete separation of the gliding parts is reached when gliding occurs in the average element.

The constant D of the secondary structure does not appear in Eq. (7) of the stress strain curve. The argument implies the existence of such a structure since we consider a certain number of blocks per unit area. The only hypothesis is that this number has to be of a different order of magnitude than that measuring the size of the specimen from which we derive the stress strain curve. As we are interested in the mean value of the stress over a macroscopic area, it is natural that the size of the blocks is not involved in the result of these calculations. To establish a relation between D and l, we have to introduce a new hypothesis and take into account a physical property of the average block. We can assume, for example, that the average block is such that its elastic deformation is of the order of magnitude of one atomic distance. In this case l would be equal to the ratio of the constants of the primary and the secondary lattice, i.e., d/D. Thence we may obtain an approximate value of D, as l is known experimentally. We shall see in the second part of this paper that the mean experimental value of l is 0.5×10^{-3}, for copper. The resulting approximate value for D is 0.8μ which is in good agreement with the value which Zwicky deduces from other considerations.

Stress strain curve of a crystal having an elastic limit

In the preceding sections we assumed that however small the tension is, a gliding occurs in a certain number of blocks. We can transform our results very easily if we admit the existence of an elastic limit, i.e., a range of deformation in which Hooke's law is applicable. The stress strain curve for which we found the equation represents the phenomenon of plastic deformation starting from the elastic limit, because at this point the first elements are ready to glide, and the conditions required by our hypotheses are present. Mathematically, if we call τ_s the elastic limit and γ_s the corresponding deformation, it is easy to see that we have to replace in Eq. (7), γ by $\gamma-\gamma_s$, τ_m by $\tau_m-\tau_s$, and add the value τ_s of the elastic limit to the result. So that the new formula is

$$\tau=\tau_s+(\tau_m-\tau_s)[1-e^{-G(\gamma-\gamma_s)/(\tau_m-\tau_s)}]. \quad (9)$$

This equation is applicable for $\tau>\tau_s$. When τ is less than τ_s, we make use of Hooke's law $\tau=G\gamma$. We must point out here that all these results are valid only for a tension inferior to the creep limit. The creep phenomenon occurs for rather small tensions in single crystals of pure metals, and in this case the stress strain curve has to be derived from a more general theory.

Possible explanation of plastic hysteresis and measure of the lost energy

The calculations made in the preceding chapter are independent of the direction of the deformation. The stress strain curve 1 (Fig. 5) may be completed by a curve 4, symmetrical with respect to the origin. This curve is obtained experimentally by taking the crystal free from tension, and applying the stress in the opposite direction.

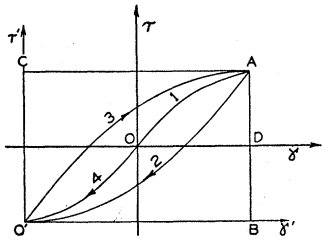

FIG. 5. Form of a hysteresis cycle.

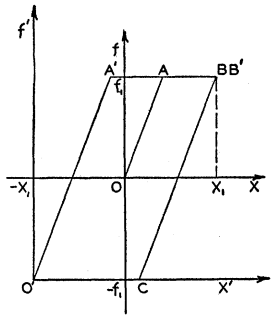

FIG. 6. Hysteresis cycle for a single element of the model Fig. 3.

If, after having deformed the crystal to the state corresponding to point A (Fig. 5), we decrease the stress and then increase it in the opposite direction until we reach the same absolute value, we obtain curve 2. The inverse process gives us curve 3 symmetrical to 2 with respect to O. We obtain a hysteresis cycle. The model which we used to explain the form of the stress strain curve gives a good explanation of the hysteresis phenomenon. The existence of a cycle can be derived (Fig. 6) by considering a *single* element of the model. The stress strain curve of this element is formed by two straight lines OA and AB if we stop the deformation x_1 at this point. If we impose on this element a deformation equal to $-x_1$, the force f will at first decrease linearly, following a straight line parallel to OA, until it reaches the symmetrical value $-f_1$. This is due to the fact that, so far, the spring alone is responsible for the deformation. As we continue to increase the deformation, gliding occurs between the two surfaces and the force remains constant until O'. The inverse process gives us the lines $O'A'$ and $A'B'$ closing up the parallelogram. If we take the axes $O'x'f'$ parallel to Oxf, we easily find that the curve $O'A'B'$ is, considered with respect to the axes $O'x'f'$, obtained by multiplying by two the coordinates of the points of the curve OAB. The two curves are similar. A very simple calculation shows that this property is also applicable in the case of n elements in the model. So if $\tau=f(\gamma)$ is the equation of the stress strain curve 1 (Fig. 5) the equation of curve 3 with respect to a system of axis $O'\gamma'\tau'$, will be $\tau'=f(\gamma')$. The correspondence between τ, τ', γ, γ' is given by $\gamma'=2\gamma$, $\tau'=2\tau$. Curves 1 and 3 are

exactly the same but the scale is changed. Therefore when in possession of the stress strain curve we may draw the hysteresis cycle corresponding to any one point A of this curve; the condition being that the deformation varies between two equal and opposite values.

Hence the calculation of the area of the cycles is very easy. The double of the area of the cycle represented in Fig. 5 is equal to the sum of the two areas $O'3AB$ and $O'2AC$, minus the area of the rectangle $ABO'C$. The area $O'3AB$ (identical to $O'2AC$) is equal to four times the area $O1AD$. Finally we have, if we denote by A the area of the cycle, and by γ and τ the coordinates of the summit of the cycle,

$$A=8\int_0^{\gamma}\tau d\gamma-4\gamma\tau.$$

In the general case, we have, between τ and γ, Eq. (9). If we replace τ by this value, the final formula will be

$$A=4(\tau_m-\tau_s)[\gamma+2(\tau_m-\tau_s)/G]e^{-G(\gamma-\gamma_s)/(\tau_m-\tau_s)}$$
$$+4\gamma\tau_m-(8/G)(\tau_m-\tau_s)^2-8\gamma_s\tau_m+4\gamma_s\tau_s.$$

The mathematical form of this exponential is

$$A=a_1(\gamma+a_2)e^{-C(\gamma-\gamma_s)}+a_3\gamma+a_4$$

in which the a_i's, and C are obtainable from the three constants τ_s, G, τ_m. Beyond a certain deformation, we can see that the term $a_3\gamma$ becomes the most important and the variation of the area with respect to the deformation is almost linear. This amounts to neglecting the curvature of the cycles and considering them as parallelograms.

PART II

The second part of this paper contains some experimental verifications of the preceding considerations. The main results needing confirmations are:

(1) The formula of the stress strain curve.

(2) The similitude between the stress strain curve and a branch of a cycle and as a consequence, the exponential law giving the area of the cycles in terms of the deformation.

The following experimental results were obtained from torsion tests made on specimens of copper of different crystalline structure. The torsion machine was built for small cylindrical specimens of about 2 mm diameter. The torque was directly measured by weights and the angle of deformation by means of the ordinary optical method. The specimens which were used for the experiments are called A, B, C and D. They have the following characteristics: A, B and C were cylinders of polycrystalline copper; the deformation was measured on a length of 10 mm. The specimen A was simply annealed by heating in vacuum a few hours in order to have it free from tension. The structure was microcrystalline. Specimens B and C were recrystallized after cold hardening, by heating in vacuum for five days at a temperature of about 1000°C. The average size of the crystals was 0.2 mm for B and 2 mm for C. Specimen D was a single crystal of copper which had been obtained by recrystallization. The copper wire was first annealed by heating two hours in vacuum at 900°C. Then it was permanently deformed about 1 percent, and heated for eight days at 1000°C. By operating on several wires, we obtained a few single crystals greater than 10 mm in length. The three specimens A, B and C were used to obtain the cycles given in Figs. 7, 8, 9. To have a series of regular cycles, it is advisable to begin with the one corresponding to the greatest stress. Having fixed the absolute value of this stress, we measure the deformations corresponding to different intermediate stresses. A very important condition which must be satisfied is that we must never change the sense of the stress before having reached the summit of the cycle. The deformations which we measure are taken with respect to the summit of the cycle as origin. The axis of deformation is fixed later as an

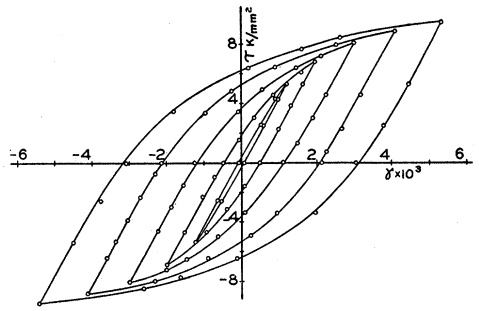

FIG. 7. Hysteresis cycles for polycrystalline copper (microcrystalline structure).

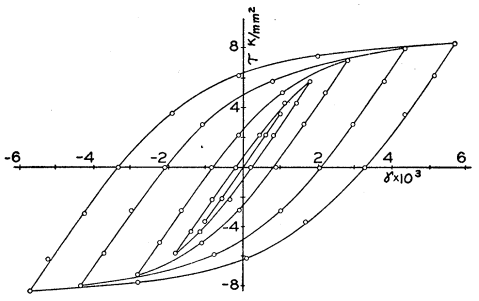

FIG. 8. Hysteresis cycles for polycrystalline copper (*B*). (Approximate size of the grain 0.2 mm.)

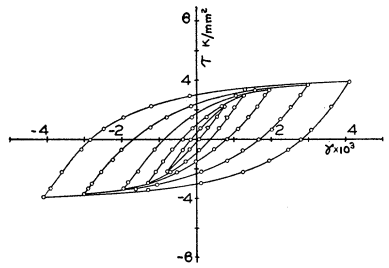

FIG. 9. Hysteresis cycles for polycrystalline copper (*C*). (Approximate size of the grain 2 mm.)

axis of symmetry of the cycle. The cycles of smaller amplitude are obtained by the same process. They are finally all drawn on the same diagram, taking into account the conditions of symmetry. The locus of the summits of the cycles of the resulting diagram gives us the stress strain curve. Having the experimental values of the constants τ_s, τ_m and G, we can calculate the stress strain curve by Eq. (9). The values of these constants and the theoretical and experimental values found for the stresses and the areas of the cycles are given in Tables I and II. The discrepancies are collected in two columns.

The single crystal (specimen *D*) gives a similar stress strain curve (see Fig. 10 and Table II). For this specimen we increased the stress until a measurable creep was reached. By measurable creep, we understand that the angular deformation γ, observed 24 hours after application of the load, increases more than 2.2×10^{-5}, which is the accuracy of the measurements. The hysteresis cycles represented in Fig. 10 are obtained as follows: We increase the stress from zero to 2.7 kg/mm² (point *A*) where we observe a creep. The speed of creep is such that the deformation reaches point *B* after 192 hours (the increase of deformation seems to follow an exponential law). Then, decreasing the stress, we reach point *C* where the creep phenomenon takes place again. After 94 hours, the deformation has increased to point *D*. We apply the inverse process, and the branch *DE* is obtained. We observe the creep between *D* and *F* for 330 hours. Finally we can obtain the hysteresis cycle *FG* without measurable creep if we do not wait more than one hour before reversing the stress at the end of the cycle.

The mechanical model considered in the former section does not take into account the phenomenon of creep. But we can easily understand that when the deformation has reached a certain value, gliding has already occurred in a great number of elements and since very few of them remain elastic, these few are alone responsible for the stability of the deformation. Practically this number is so small that creep occurs. After a certain amount of this creep, there is no

TABLE I. *Elastic limits, deformations and constants.*

SPECIMEN	τ_s (kg/mm²)	$\gamma_s\times10^5$	τ_m (kg/mm²)	G (kg/mm²)
A	3.0	66	9.5	4550
B	3.15	87	8.65	3620
C	0.6	18	4.0	3380
D	1.0	31	2.9	3180

TABLE II. *Theoretical and experimental values of the stresses of the hysteresis cycles.*

SPECIMEN	$\gamma\times10^5$	THEOR. STRESS (kg/mm²)	EXP. STRESS (kg/mm²)	DIFF.	THEOR. AREA (kg/mm²)	EXP. AREA (kg/mm²)	DIFF.
	66.0	—	—	—	—	—	—
	120.0	5.07	5.09	+0.02	0.0008	0.0008	0
A	197.2	6.90	6.87	−0.03	0.0089	0.0094	+0.0005
	299.5	8.22	8.06	−0.16	0.0267	0.0276	+0.0009
	410.0	8.90	8.82	−0.08	0.0595	0.0572	−0.0023
	540.0	9.25	9.48	+0.23	0.0964	0.0936	−0.0029
	101.7	3.67	3.62	−0.05	0.0002	0.0000	−0.0002
	176.2	5.60	5.84	+0.24	0.0040	0.0037	−0.0003
B	281.0	7.12	7.25	+0.13	0.0172	0.0174	+0.0002
	434.0	8.10	7.98	−0.12	0.0549	0.0538	−0.0011
	569.0	8.31	8.35	+0.04	0.0889	0.0906	+0.0017
	78.6	2.14	2.22	+0.08	0.0007	0.0005	−0.0002
	132.0	2.93	2.96	+0.03	0.0028	0.0032	+0.0004
C	195.0	3.40	3.34	−0.06	0.0080	0.0084	+0.0004
	298.2	3.79	3.71	−0.08	0.0191	0.0192	+0.0001
	410.0	3.93	3.90	−0.03	0.0347	0.0353	+0.0006
	37.8	1.19	1.20	+0.01			
	47.1	1.44	1.44	0.00			
	56.3	1.65	1.68	+0.03			
D	67.8	1.87	1.93	+0.06			
	82.7	2.09	2.16	+0.07			
	110.0	2.39	2.41	0.02			
	126.5	2.51	2.53	+0.02			
	162.0	2.68	2.65	−0.03			

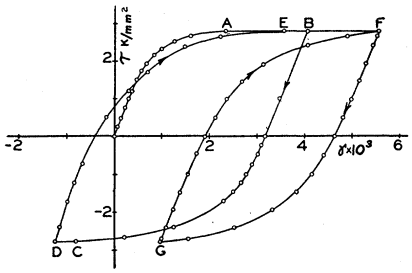

FIG. 10. Hysteresis cycles for a copper single crystal (D) in torsion.

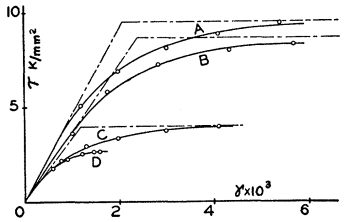

FIG. 11. Stress strain curves in torsion for polycrystalline copper (A, B and C) and a single crystal of copper (D).

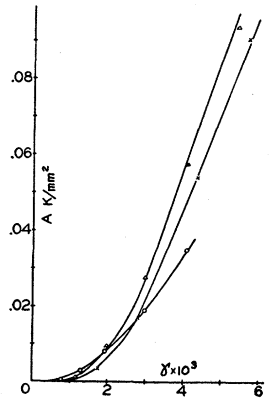

FIG. 12. Areas of the hysteresis cycles in function of the deformation (A, B and C).

reason at all why we could not reproduce a hysteresis cycle identical to the cycle before creep. This is proved experimentally by the fact that the branches of the different cycles drawn in Fig. 10 are identical.

Figs. 11 and 12 show graphically the experimental verifications.

CONCLUDING REMARKS

The experimental results shown in Table II confirm the theory. In addition to the two main results which confirm the theoretical conclusions (1) and (2) stated at the beginning of Part II, we can point out that the tangent at the end of each cycle has always a slope equal to the modulus of elasticity. More generally, each time we reverse the sense of the deformation, we obtained an inclined straight line, if the elastic limit was different from zero. This can be deduced from the theory.

This study does not constitute a direct proof of the existence of a secondary structure in crystals. However, the experimental verifications agree so well with the calculations based on the existence of a secondary structure that it gives a proof *a posteriori* of the reality of such a structure. Our mechanical model is only a model, and we do not maintain that gliding along a π-plane is identical to that of two material surfaces. But, however complicated the motion of the particles in the π-plane may be, the final situation of the two atomic rows is about the same as that which we would obtain by simple gliding.

We can also point out the great regularity which we found in the form of the hysteresis curves obtained with four different specimens. We think that this regularity could be extended to other crystals. And this seems to prove that there is something fundamental in the structure of any crystal which is responsible for the plastic deformation. The notion of a secondary structure has this character of something fundamental and provides a very good tool for systematic researches on the mechanical properties of crystals.

In our experiments we have used the method of torsion as it is the simplest way of obtaining states of pure shearing stress. However, this stress is not uniform over the cross section of the wires. We therefore intend to conduct further experiments on crystals which are subjected to uniform alternating shearing stresses in order to reduce the problem of mechanical hysteresis to the simplest possible case.

In conclusion I wish to express my gratitude to Professors Th. von Kármán and F. Zwicky, under whose direction this investigation was carried out. I am indebted to them for frequent and valuable advice.